Journal of Science and Education (JSE)
Vol 6, Issue 2, March 2026, Pages 1263-1275
ISSN: 2745-5351 (Media Online)
DOI: https://doi.org/10.58905/jse.v6i2.737

Profile of Students' Ability to Solve Group Material Problems Based on APOS Theory Reviewed from Differences in Mathematical Ability

Fatriya Adamura[1[0000-0001-7510-4275]]*, Dwi Juniati[2[0000-0002-5352-3708]], Siti Khabibah[2[0000-0002-1292-0619]]

[1]Universitas PGRI Madiun, Madiun, Indonesia
[2]Universitas Negeri Surabaya, Surabaya, Indonesia

*Corresponding Author: fatriya.mathedu@unipma.ac.id

Abstract. This study aims to describe the profile of students' ability to solve group material problems based on APOS theory reviewed from differences in mathematical ability. The research uses a qualitative descriptive approach that focuses on revealing students' thinking processes in understanding abstract algebraic concepts. The research subjects consisted of three students who were purposively selected based on the results of the mathematical ability test, namely students with high, medium, and low mathematical ability. The research instruments include mathematical ability tests, group material problem solving tests designed according to the stages of APOS theory, and semistructured interview guidelines. Data is analyzed through the stages of data reduction, data presentation, and conclusion drawing by referring to the indicators of each stage of APOS, namely action, process, object, and schema. The results of the study show that students with high mathematical ability are able to fulfill all stages of APOS completely and consistently. Medium-skilled students are able to reach the action, process, and object stages, but have not fully reached the schema stage. Meanwhile, low-skilled students are only able to perform part of the action stage and experience difficulties at the process, object, and schema stages. These findings show that early mathematical ability has a significant effect on the construction of group concept comprehension. This study confirms that APOS theory is effectively used as an analytical framework to map students' conceptual understanding of abstract algebraic materials, especially group concepts, and provides important implications for the design of mathematics learning in higher education.

Keywords: Ability Profile, APOS, Group Material, Math skills, Students

1 Introduction

Abstract algebra is one of the core courses in the mathematics and pure mathematics education study program that requires a high level of abstract thinking skills. One of the fundamental materials in abstract algebra is the concept of groups that includes binary operations, identity elements, inverses, and associative properties. Group materials emphasize not only procedural skills, but also conceptual understanding and mathematical proof skills. Understanding abstract concepts in mathematics requires a complex mental restructuring [1]. APOS (Action, Process, Object, Schema) theory provides a cognitive framework to understand how students build mathematical concepts gradually. This theory emphasizes that conceptual understanding evolves from procedural action to an organized conceptual structure [2]. Therefore, APOS theory is relevant to analyze students' understanding of group material. The analysis of student ability profiles through APOS theory is expected to provide a comprehensive picture of students' mathematical thinking processes.

In learning practice, many students have difficulty in solving group material problems, especially those related to proving and generalizing concepts. Students tend to memorize definitions without being able to relate them to a broader mathematical structure. The main difficulty students in abstract algebra lies in the transition from procedural to conceptual understanding [3]. This indicates that students often stop at the action or process stage and have not reached the object or schema stage. This problem is increasingly complex because students have diverse initial math skills. These differences in abilities affect the way students process and solve problems. However, classroom learning often does not accommodate these differences optimally. As a result, some students are left behind in understanding the concept of the group in its entirety.

Various studies have examined students' difficulties in abstract algebra, but most have focused only on learning outcomes or misconceptions. There are still few studies that specifically map the profile of students' abilities based on the cognitive stages of APOS theory in group materials. In addition, research linking APOS theory to differences in students' mathematical abilities is still limited. The understanding of mathematical concepts is greatly influenced by the cognitive readiness of the individual [4]. Another gap found was the lack of in-depth descriptions of how students with high and low abilities build group concepts. Most studies use a quantitative approach without exploring students' thought processes. Therefore, research that combines cognitive analysis and variation in mathematical abilities is needed. This study seeks to fill this gap through APOS-based capability profile analysis.

As an alternative solution, APOS theory can be used as an analytical framework to identify the stages of student understanding in solving group material problems. Using this theory, lecturers and researchers can find out whether students are at the action, process, object, or schema stage. In addition, the grouping of students based on mathematical ability allows for a more specific and fair analysis. APOS stage mapping can be the basis for designing more effective learning. Ability profile analysis also allows for the delivery of targeted learning interventions [5]. Students with low abilities can be facilitated to transition to a higher cognitive stage. Meanwhile, students with high abilities can be given challenging tasks to reinforce the conceptual scheme. Thus, this approach is both diagnostic and solution-oriented.

The theory of APOS is appropriately placed as an analytical framework due to its characteristics that emphasize the gradual construction of mathematical concepts from procedural to mature cognitive structures. In the context of highly abstract group

Journal of Science and Education (JSE)
Vol 6, Issue 2, March 2026, Pages 1263-1275
ISSN: 2745-5351 (Media Online)
DOI: https://doi.org/10.58905/jse.v6i2.737

material, APOS provides a strong theoretical basis for explaining how students build an understanding of binary operations, identities, inverses, and group structures as a whole. The authors justify the use of APOS by referring to the cutting-edge literature that asserts that understanding abstract algebraic concepts demands a mental reconstruction that evolves from action to schema. The position of APOS in the manuscript is appropriate and logical as a tool to map the development of students' understanding in depth.

Research on the application of APOS theory in mathematics learning has developed in recent years. APOS theory is effective in improving students' understanding of concepts in calculus materials [6]. APOS analysis is able to reveal students' conceptual difficulties in proving mathematics [7]. APOS to analyze problem-solving and obtain a detailed overview of the student's thinking process [8]. However, most of the research has not focused on group material. In addition, the variables of initial mathematical ability are rarely used as the basis for the analysis of cognitive profiles. In fact, initial abilities have a significant effect on the way students build concepts. Therefore, this study continues and expands on previous findings.

The novelty of this research lies in the mapping of the profile of students' ability to solve group material problems based on APOS theory which is reviewed from differences in mathematical ability. This research not only assesses the final results, but also reveals the students' thought process in depth. The integration between APOS analysis and the grouping of mathematical abilities provides a new perspective in the study of abstract algebra [9]. In addition, this study presents a description of the characteristics of each ability group at each stage of APOS. These findings can be used as the basis for the development of group learning diagnostic instruments. In contrast to previous research, the focus of this study is on cognitive profiles, not on the effectiveness of learning models. Thus, this research makes a simultaneous theoretical and practical contribution. This novelty is expected to enrich the literature of mathematics education at the university level. The article convincingly identifies a research gap, namely the lack of studies that integrate APOS theory with differences in students' mathematical abilities in the context of group material. Previous research has focused more on learning calculus, limits, or error analysis without mapping APOS-based cognitive profiles in depth in group material. The authors point out that few studies have simultaneously analyzed the stages of APOS and students' initial mathematical ability in understanding group structure. Therefore, the manuscript offers a new contribution in the form of a *comparative cognitive profile* between high, medium, and low-ability students in understanding the concept of groups.

Recent research developments show a growing interest in cognitive analysis in advanced mathematics learning. APOS theory is one of the dominant approaches to understand the construction of abstract concepts of students. Recent research emphasizes the importance of thinking process analysis rather than just measuring learning outcomes [10]. On the other hand, the study of abstract algebra highlights the need for a more conceptual and reflective pedagogical approach. Research trends also show the use of qualitative methods to explore student understanding in depth. However, integration between APOS, group material, and differences in math abilities is still rare.

Therefore, this research is at the forefront (state of the art) of mathematics education studies. This study bridges cognitive theory and abstract algebraic learning practice.

The urgency of this research is based on the importance of group material as a foundation for advanced mathematics courses. Lack of understanding of this material can have an impact on learning difficulties at the next level. In addition, the demands of 21st-century learning emphasize mastery of high-level thinking and conceptual understanding. Without a clear ability profile mapping, it is difficult for lecturers to design learning that suits student needs [11]. The difference in students' mathematical abilities also demands an adaptive learning approach. This research provides the empirical data needed to design differential learning strategies. From the academic side, the results of the research can be a reference for curriculum development. Therefore, this research has a high theoretical and practical urgency.

The manuscript has articulated the focus of the research clearly, namely describing the profile of students' ability to solve group theory problems based on cognitive stages in APOS theory, as well as reviewing the differences in students' initial mathematical abilities. The relationship between *difficulties in understanding group concepts* such as difficulty proving the nature of the group, transitions from procedural to conceptual, and misconceptions about identity or inverse is explicitly integrated with the APOS stage. The article shows that many difficulties arise because students stop at the *action* and *process* stages, so they have not reached *the object* and *schema*. Thus, the focus of the research is not only on the final outcome, but on the *cognitive flow* that causes variations in students' understanding based on their mathematical abilities.

The general purpose of this study is to describe the profile of students' ability to solve group material problems based on APOS theory reviewed from differences in mathematical ability. In particular, this study aims to identify the stages of APOS that arise in the solution of group material problems. This study also aims to compare the ability profile of students with high and low mathematical ability. In addition, this study aims to uncover the characteristics of student difficulties at each stage of APOS. The results of the research are expected to provide a comprehensive overview of the students' thinking process. The findings of this study also aim to be the basis for the development of more effective abstract algebra learning. Thus, this research contributes to improving the quality of mathematics learning in universities. This goal is in line with the needs of developing cognitive-based mathematics education.

2 Method

This research is a descriptive research with a qualitative approach. The selection of this type of research aims to describe in detail the characteristics and conditions of the research subject in accordance with the facts found in the field. Descriptive research is used because this research does not aim to test hypotheses, but rather to explain phenomena that occur in the research subject. The qualitative approach allows researchers to gain a deeper understanding of students' thinking processes in solving mathematical problems. This research was carried out in the odd semester of the 2025/2026 school year. The location of the research is PGRI Madiun University. The research subjects

Journal of Science and Education (JSE)
Vol 6, Issue 2, March 2026, Pages 1263-1275
ISSN: 2745-5351 (Media Online)
DOI: https://doi.org/10.58905/jse.v6i2.737

came from 7th semester students. The focus of the research is directed at the analysis of students' abilities based on variations in mathematical abilities. The purpose of the research is to profile students' ability to solve group material problems based on the APOS stage has been coherently aligned with the method used. Mathematical ability test instruments, APOS-based problem-solving tests, and semi-structured interviews support the goal of uncovering the subject's thought process in depth. The analysis procedure carried out starting from the identification of *actions*, *processes*, *objects*, to *schemas* directly refers to the research objective, namely mapping students' cognitive development. Thus, there is consistency between the objectives, data collection methods, and APOS analytical framework so that the approach used is able to produce a valid and targeted profile of students' abilities.

The research subjects consisted of three students who were selected to represent different levels of mathematical ability, namely high, medium, and low abilities. The selection of subjects aims to obtain an overview of the diverse ability profiles in solving mathematics problems. Each subject is chosen as a representation of a specific group of abilities. With a limited number of subjects, the study placed more emphasis on the depth of analysis than the number of participants. The subject selection technique is carried out purposively, which is based on predetermined criteria. The researcher selects subjects that are considered capable of providing rich and relevant data. This strategy is in line with the characteristics of qualitative research. Thus, the chosen subjects are expected to provide a clear picture of the student's ability profile.

The subject selection process began with the provision of a mathematical ability test to all 7th semester students totaling 30 people. This test is used to measure students' initial math skills. The results of the test are then analyzed to determine the category of students' mathematical ability. Based on the results of the analysis, students are grouped into three categories, namely high ability, medium ability, and low ability. From each of these categories, one student was selected as a research subject. The selection is carried out by considering the completeness of the answers and the ability of students to express their thinking process. After the subjects were selected, they were given a follow-up test in the form of a problem-solving test. This test is used to delve deeper into students' ability to solve math problems.

After the subject completes the problem-solving test, the next stage is the interview. The interview was conducted to obtain additional information about the reasons, strategies, and ways of thinking of students that were not entirely apparent from the written answers. Interviews also serve to confirm and strengthen the data obtained from the results of the written test. Thus, the data collected becomes more complete and accurate. Interviews were conducted individually to each subject. The interview process is carried out after students complete the written test. All interview activities are recorded to facilitate the data analysis process. The recorded data is then transcribed for further analysis.

The instruments used in this study included mathematical ability tests, problem-solving tests, and interview guidelines. Mathematical ability tests are used as a tool to determine the subject of research. The questions in the mathematics ability test are prepared by adapting group material questions that are relevant to the 7th semester student material. The form of the question is changed to a description question so that students

Journal of Science and Education (JSE)
Vol 6, Issue 2, March 2026, Pages 1263-1275
ISSN: 2745-5351 (Media Online)
DOI: https://doi.org/10.58905/jse.v6i2.737

can show their thinking process more clearly. Each question item is designed to measure students' understanding of concepts and procedural skills. The math ability test consists of 3 description questions. The selection of the form of the description question aims to avoid the possibility of students answering by guessing. Thus, test results reflect the student's true mathematical ability.

The level of math proficiency was defined through an initial ability test given to 30 students, then classified into three categories of high, medium, and low based on the score range (≥86 high, 66–85 medium, <66 low). This measurement procedure is quite strict because the assessment is carried out in a structured manner with instruments in the form of description questions that reveal mastery of concepts, not just multiple-choice results. The conceptualization of mathematical ability is also justified because the study wants to see how initial abilities affect the ability to reach the stages of APOS in solving group problems. As such, this categorization is relevant and has a clear methodological basis.

The problem solving test was used to reveal the profile of students' ability to solve group material problems based on apos theory reviewed from differences in mathematical ability. This test consists of three description questions arranged according to the stages in the APOS theory. The first question part (a) is used to identify students' abilities at the action stage. The first question part (b) is designed to see the ability of students at the process stage. The second question is used to reveal students' abilities at the object level. The third question is a question about the application of the quadratic function which aims to determine the ability of students at the scheme stage. With this arrangement, each stage of APOS can be analyzed systematically. This test allows researchers to map the gradual development of students' understanding.

The interviews used in this study were semi-structured interviews. The interview is conducted using pre-arranged guidelines, but still provides room for the subject to develop his answers. This approach allows researchers to obtain more in-depth and flexible information. The interviews were focused on clarifying students' written answers and exploring their thought processes. During the interview, the researcher provides follow-up questions according to the subject's response. The entire interview process was recorded using a voice recording device. The recording results are then transcribed verbatim. Interview transcripts are used as supporting data in the analysis.

Data analysis of the results of the mathematical ability test was carried out by matching students' answers with the answer keys that had been compiled by the researcher. Each student's answer is analyzed to determine the level of mastery of the material. Based on the results of the analysis, students are grouped into high, medium, and low ability categories. This grouping refers to criteria that have been established in the form of a classification table of abilities. Furthermore, data from the question solving test and interviews were analyzed qualitatively. The analysis was carried out by associating student answers with APOS stage indicators. The results of the analysis are presented in the form of narrative descriptions. The analysis process is carried out repeatedly to ensure the accuracy and consistency of the findings.

Journal of Science and Education (JSE)
Vol 6, Issue 2, March 2026, Pages 1263-1275
ISSN: 2745-5351 (Media Online)
DOI: https://doi.org/10.58905/jse.v6i2.737

Table 1. Math Ability Criteria

Categories of Math Ability	Test Score Range
Height	Test score ≥ 86
Medium	66 ≤ test score < 86
Low	Test score < 66

Data analysis in the question solving test was carried out by referring to the answer key that had been compiled by the researcher. Each student's answer is analyzed to see the accuracy of the steps, the use of concepts, and the reasons given in solving the questions. The results of this analysis are used to identify the suitability of students' answers with the indicators that have been set. In describing the ability of students to solve group material problems based on APOS theory reviewed from differences in mathematical skills, an analysis table is used that contains indicators at each stage of APOS, namely actions, processes, objects, and schemes. The table serves as a guideline in classifying students' answers according to the stage of development of understanding mathematical concepts. Using this table, researchers can map the profile of students' abilities systematically and consistently. The classification criteria at each stage of APOS are explicitly stated through indicators of actions, processes, objects, and schemes, and are directly related to the characteristics of students' responses in solving group material problems. The validity of the classification is strengthened through triangulation between the results of the student's work (written work) and the interview that confirms the reason and completion strategy. For example, students who are able to explain processes without relying on examples are positioned at *the process* stage, while those who can relate the properties of groups as a single unit are placed at the *object* stage. This combination of written and verbal data makes the classification of APOS in research quite reliable, consistent, and evidence-based.

3 Results and Discussion

3.1 Results

The data collection of this research will be carried out in November 2025. Data collection began with the provision of a mathematics ability test to all 7th semester students totaling 30 people. Based on the results of the test, students were grouped into three categories of math ability, namely high, medium, and low. From each of these categories, one student was selected as a research subject. Thus, three students were obtained who represented high, medium, and low mathematical abilities. The selection of subjects is carried out based on the results of students' work on the math ability test. The selected subjects were then further analyzed in this study. The three subjects are considered to be able to represent the characteristics of each category of ability. The data of the selected research subjects are presented in table 2 below.

Journal of Science and Education (JSE)
Vol 6, Issue 2, March 2026, Pages 1263-1275
ISSN: 2745-5351 (Media Online)
DOI: https://doi.org/10.58905/jse.v6i2.737

Table 2. Research Subject Data

No	Name	Score	Categories
1	Riani	95	Height
2	Rahel	84	Medium
3	Al Rohma	65	Low

Furthermore, to simplify the analysis process and maintain the confidentiality of the subject's identity, each subject is assigned a code. Subjects with high math ability are coded ST, subjects with moderate math abilities are coded SS, and subjects with low math abilities are coded SR. The use of this code is carried out consistently throughout the data analysis stage. In the next section, a summary of the results of the question completion test and interview from each subject, namely ST, SS, and SR will be presented. This presentation aims to describe in detail the profile of the subject's ability to solve problems based on the APOS theory.

The following will be presented a summary of the results of the interview and the subject's work in writing from ST, SS, and SR subjects.

Subjects with High Mathematical Ability (ST)

The ST subject is able to correctly define and verify the properties of a given set and binary operation as a candidate group. In solving problems, ST can systematically examine closed, associative, the existence of identity elements, and inverse elements. ST does not use only one procedure, but is able to apply more than one method, for example by using formal definitions of groups and by utilizing the properties of algebra that have been learned. This flexibility shows that ST has thoroughly understood the basic procedures. This indicates that ST has met all indicators at the action stage.

At the process stage, ST is able to explain the steps to prove that a structure meets the axioms of the group in a logical and logical manner. ST can associate each proof step with the relevant group definition. In addition, ST is able to generalize the process carried out, for example by explaining why an operation fulfills associative properties without having to calculate one by one. This understanding shows that ST has internalized his thought process. Thus, all process stage indicators have been fulfilled by ST.

At the object level, ST is able to view the concept of groups as a unit of mathematical objects. ST can identify examples and non-examples of groups along with the exact reasons based on the group's axioms. In addition, ST is able to distinguish groups from other algebraic structures, such as semigroups or monoids. This shows that ST has constructed the concept of groups as abstract objects. Thus, all object level indicators have been met by ST.

At the schema stage, ST is able to solve group concept application problems, such as defining subgroups, examining groups in modular contexts, or relating group concepts to advanced math problems. ST can integrate various concepts related to the group in one complete solution. This ability shows that ST has built a mature conceptual scheme. Therefore, all indicators of the scheme stage have been fulfilled by ST.

Subject with Moderate Mathematics Ability (SS)

The SS subject is able to determine whether a set with a particular operation meets most of the group's requirements. SS can check the closed nature and correctly determine the identity elements as well as inverses. In solving the problem, the SS uses a procedure that corresponds to the definition of the group, although it still follows the steps mechanically. SS is also able to solve problems in more than one way in certain cases. This shows that the SS has met the action stage indicators.

At the process stage, SS is able to explain the steps to prove that a structure is a group, although the explanation given is not completely in-depth. SS still relies on concrete examples to explain the process carried out. However, the sequence of steps that has been arranged has shown a fairly good understanding. Therefore, it can be said that SS has met most of the process stage indicators.

At the object stage, SS is able to identify examples of groups and not groups based on formal definitions. The SS is also able to mention the general form of the group structure. However, the reasons given are sometimes still descriptive and not completely conceptual. This shows that SS's understanding of the concept of groups as mathematical objects has been established, but not yet fully strong. Thus, the object-level indicator has been met, although there are still limitations.

At the schema stage, SS has not been able to fully solve the application of the group concept. SS can design the initial steps of the solution, but it has not been able to integrate the various concepts of the group as a whole until the final solution is correct. This shows that the SS has not reached the optimal scheme stage. Therefore, not all indicators of the scheme stage are met by SS.

Low Mathematics Ability Subject (SR)

The SR subjects were able to examine some basic conditions of the group, such as determining the identity element or inverse, with the correct results. However, the strategies used are still procedural and less efficient. SRs tend to follow one memorized way of completion without considering other alternatives. However, mathematically the steps used are still acceptable. This shows that SR has met the action stage indicators, although its understanding is still limited.

At the process stage, SR had difficulty in explaining the steps of proof in a sequential manner. SR has not been able to clearly relate each step to the definition or nature of the group. The process carried out is still partial and inconsistent. Therefore, not all process stage indicators can be met by SR.

At the object level, SR is able to recognize that a certain structure is related to the concept of a group, but is not yet able to provide a precise reason. SR also has difficulty in conceptually distinguishing between group and non-group examples. This shows that the concept of groups has not yet been fully formed as an abstract object in the understanding of SR. Thus, the object-stage indicator has not been optimally met.

At the schema stage, SR has not been able to solve the application of the group concept. SR cannot integrate the various properties and concepts of the group in one complete solution. This shows that SR has not yet reached the schematic stage in the APOS theory. Therefore, all indicators of the scheme stage have not been fulfilled by SR.

Journal of Science and Education (JSE)
Vol 6, Issue 2, March 2026, Pages 1263-1275
ISSN: 2745-5351 (Media Online)
DOI: https://doi.org/10.58905/jse.v6i2.737

3.2 Discussion

The manuscript provides a clear and rich comparative analysis between high, medium, and low-ability students, especially regarding the difference in APOS stage achievement and the quality of group concept understanding. High-ability students (ST) are shown to be able to reach all stages of APOS, while moderately capable students only reach objects and low-ability students even have difficulties in the process. This comparison is strengthened by concrete evidence in the form of problem-solving patterns, the ability to build arguments, how to verify identity or inverse, and interview narratives. Therefore, the comparative analysis is presented meaningfully and provides an in-depth understanding of how early abilities affect APOS' cognitive development.

The results of the study show that there is a clear difference in the profile of students' ability to solve group material problems when viewed from the theory of APOS and differences in mathematical ability. Highly capable students tend to be able to achieve the four stages of APOS, namely action, process, object, and schema. These findings are in line with the view [12] which states that the understanding of abstract concepts is formed through a tiered mental transformation. Students with high mathematical skills are able to perform procedural actions flexibly and internalize them into meaningful processes. This suggests that cognitive readiness plays an important role in the formation of conceptual schemas. On the other hand, students with medium and low ability show limitations at a certain stage. This condition shows that not all students are able to make optimal cognitive transitions. Thus, the APOS theory has proven to be relevant for mapping the variation in understanding of group concepts in students.

The study did not stop at the identification of errors or the achievement of results, but provided an in-depth cognitive analysis at each stage of APOS for each subject. The author describes how students in each category of mathematical ability execute actions, internalize processes, build objects, and integrate schemes. The interview data also strengthens the analysis by showing the reasons, strategies, and mindsets that students use when solving problems. As a result, the research was able to explain *why* students fail or succeed at a certain stage, not just *what* went wrong. This shows a strong depth of analysis and is consistent with a qualitative approach based on cognitive theory.

At the action stage, all research subjects are able to carry out basic procedures related to the concept of the group, such as examining the basic requirements of the group. However, the quality of the actions performed differs between the levels of mathematical ability. High-ability and moderate-ability students are able to use more than one procedure, while low-ability students tend to be fixated on one way. According to [13] The action stage is characterized by procedural activities that are still external and depend on explicit instructions. The findings of this study support this theory, especially in low-ability subjects who still rely on mechanical steps. Highly capable students show flexibility at this stage, which is the basis for the transition to the process stage. This indicates that the quality of the action affects the success of the next stage. Therefore, strengthening the action stage is important in learning group material.

At the process stage, the difference in students' abilities is increasingly apparent. Highly capable students are able to explain the steps of group proof in a concise and logical manner without relying on concrete examples. Students with moderate abilities

Journal of Science and Education (JSE)
Vol 6, Issue 2, March 2026, Pages 1263-1275
ISSN: 2745-5351 (Media Online)
DOI: https://doi.org/10.58905/jse.v6i2.737

still need illustrations or special cases to explain the process carried out. Meanwhile, low-ability students have difficulty internalizing procedures into meaningful processes. According to [14] Failure at the process stage is often caused by the inability to internalize the action into a stable mental structure. These findings show that low-ability students have not fully built a relational understanding of the concept of groups. This condition causes difficulties in compiling coherent mathematical arguments. Thus, the process stage becomes a critical point in learning abstract algebra.

At the object level, high-skilled students have been able to view the group as a complete abstract entity. This is evident from their ability to identify examples and non-group examples and distinguish them from other algebraic structures. Capable students are showing sufficient understanding of the object, but the reasons given are still descriptive. In contrast, low-ability students do not fully understand the concept of groups as mathematical objects. The object stage is reached when the process can be treated as a mental unit [15], [16]. The findings of this study show that the achievement of the object stage is greatly influenced by initial mathematical ability. The inability to build objects makes it difficult for students to step into the schema stage. Therefore, learning needs to emphasize conceptual conceptual reconstruction, not just procedural.

At the schema stage, only high-skilled students are able to integrate various group concepts to solve application problems. Moderately capable students are only able to compile a completion plan without completing it completely. Meanwhile, low-ability students are not able to relate the concept of group to more complex problems. According to [17], [18] schemas are formed when various objects and processes are interconnected in a coherent structure of knowledge. These findings confirm that the schematic stage is the highest and most complex stage in the APOS theory. Failure to reach this stage shows the weak integration of the concept. It also indicates that learning group material requires the right time and strategy. Thus, not all students automatically reach the scheme stage without adequate pedagogical intervention.

The results of this study are consistent with previous research that states that APOS theory is effective for analyzing the understanding of advanced mathematical concepts. Research [19], [20] shows that students with high mathematical ability reach the object and schema stage faster. The findings of this study reinforce these results in the context of the group material. In addition, this study confirms that differences in early mathematics skills have a significant effect on the construction of students' knowledge. The contribution of this research lies in mapping the profile of APOS students in detail in the group material. The implications of these findings suggest the need for abstract algebraic learning that is oriented towards the development of cognitive stages. Lecturers need to design activities that encourage the transition from action to process and beyond. Thus, APOS theory can be used as a foundation for the development of group material learning in higher education.

The pedagogical implications drawn from the findings are quite concrete, especially on the need for differentiatory and diagnostic learning approaches in the abstract algebra classroom. The results showed that students with low abilities needed interventions that strengthened the transition from *action* to *process*, for example with gradual exercises, conceptual scaffolding, and the use of examples. Meanwhile, high skilled students can be assigned tasks that encourage concept integration at the *schema* stage such

Journal of Science and Education (JSE)
Vol 6, Issue 2, March 2026, Pages 1263-1275
ISSN: 2745-5351 (Media Online)
DOI: https://doi.org/10.58905/jse.v6i2.737

as advanced application or abstract proof. Thus, the implications of the research directly support the design of abstract algebraic learning that is more adaptive and based on the cognitive development of students.

4 Conclusion

This study shows that students' ability to solve group material problems varies based on APOS theory and the level of mathematical ability. High-skilled students are able to achieve all stages of APOS, namely actions, processes, objects, and schemas in their entirety. Moderately capable students are generally able to reach the action, process, and object stages, but have not fully reached the schema stage. Meanwhile, low-ability students are only able to fulfill part of the APOS stage and experience difficulties in the process, object, and schema stages. The results of the study confirm that early mathematical skills have a significant effect on the construction of group concept understanding. APOS theory has proven to be effective in mapping the profile of students' conceptual understanding in abstract algebra material. Therefore, group material learning needs to be designed in stages and oriented towards the development of students' cognitive structure. This research makes a significant contribution to the field of mathematics education at the university level, both theoretically and practically. Theoretically, the research extends the use of APOS theory to the realm of abstract algebra, especially to the concept of groups that were previously more applied to calculus or limits. Practically, this study produces a detailed cognitive profile that can be used as the basis for instructional design, diagnostic assessment, and curriculum development on algebraic structure materials. The study also adds a new perspective on how early mathematical abilities interact with APOS cognitive development, thus enriching the empirical literature on abstract concept learning.

References

[1] Y. Safari and P. Nurhida, "Pentingnya Pemahaman Konsep Dasar Matematika dalam Pembelajaran Matematika," *Karimah Tauhid*, vol. 3, no. 9, pp. 9817–9824, Sept. 2024, doi: 10.30997/karimahtauhid.v3i9.14625.

[2] E. L. Langi', D. Juniati, and A. Abadi, "Reconstruction of Derivatives Concept by Prospective Teachers Based on APOS Theory Reviewed from Gender Differences," presented at the MISEIC 2019, 2019. Accessed: Nov. 27, 2025. [Online]. Available: https://miseic.conference.unesa.ac.id/index.php/ocs/miseic2019/paper/view/2173

[3] Y. M. Cholily *et al.*, *PEMBELAJARAN ALJABAR DI SEKOLAH*. Malang: UMMPress, 2025.

[4] M. Nurhangesti and Seruni, "FAKTOR-FAKTOR PEMAHAMAN KONSEP MATEMATIKA: KAJIAN LITERATUR," *Jurnal Media Akademik (JMA)*, vol. 2, no. 12, Dec. 2024, doi: 10.62281/v2i12.1381.

[5] S. Sutama, W. D. P. M.Pd, N. F. M.Pd S. Pd, and M. N. M.Pd, *DESAIN PEMBELAJARAN BERORIENTASI LITERASI NUMERASI SEKOLAH DASAR*. Surakarta: Muhammadiyah University Press, 2022.

[6] V. Borji, H. Alamolhodaei, and F. Radmehr, “Application of the APOS-ACE Theory to improve Students’ Graphical Understanding of Derivative,” *EURASIA J Math Sci Tech Ed*, vol. 14, no. 7, pp. 2947–2967, May 2018, doi: 10.29333/ejmste/91451.

[7] M. G. Baye, M. A. Ayele, and T. E. Wondimuneh, “Implementing GeoGebra integrated with multi-teaching approaches guided by the APOS theory to enhance students’ conceptual understanding of limit in Ethiopian Universities,” *Heliyon*, vol. 7, no. 5, p. e07012, May 2021, doi: 10.1016/j.heliyon.2021.e07012.

[8] N. T. Nga *et al.*, “The Effectiveness of Teaching Derivatives in Vietnamese High Schools Using APOS Theory and ACE Learning Cycle.,” *European Journal of Educational Research*, vol. 12, no. 1, p. 507, Jan. 2023, doi: 10.12973/eu-jer.12.1.507.

[9] K. U. Z. Nugroho *et al.*, *PROBLEMATIKA DAN SOLUSI DALAM PEMBELAJARAN GEOMETRI NON-EUCLIDE*. Jakarta: Feniks Muda Sejahtera, 2025.

[10] M. D. Prayoga, W. Sasmita, and A. Mahendra, “Pembelajaran Mendalam : Penekanan Pada Proses Pembelajaran Untuk Meningkatkan Hasil Penilaian Belajar Siswa,” *Philosophiamundi*, vol. 3, no. 3, pp. 548–554, July 2025.

[11] Nurdini *et al.*, *Transformasi Pembelajaran di Era Kurikulum Merdeka Belajar*. Banten: Sada Kurnia Pustaka, 2024.

[12] N. W. Pratomo, “PENGARUH PERSEPSI SISWA ATAS KOMPETENSI GURU DAN MOTIVASI BELAJAR TERHADAP PRESTASI BELAJAR BAHASA INDONESIA,” *Jurnal MANDIRI: Ilmu Pengetahuan, Seni, dan Teknologi*, vol. 2, no. 2, pp. 299–313, Dec. 2018, doi: 10.33753/mandiri.v2i2.45.

[13] J. Blegur, *Soft Skills untuk Prestasi Belajar: Disiplin Percaya diri Konsep diri akademik Penetapan tujuan Tanggung jawab Komitmen Kontrol diri*. Surabaya: Scopindo Media Pustaka, 2020.

[14] A. P. Munthe, J. N. Butarbutar, C. R. Simanjuntak, C. A. Sipayung, F. Siburian, and D. Naibaho, “DINAMIKA PSIKOLOGI PERKEMBANGAN PADA FASE PERKEMBANGAN MANUSIA DI DESA MULARAWI,” *Jurnal Ilmiah Multidisiplin Ilmu*, vol. 1, no. 3, pp. 41–49, May 2024, doi: 10.69714/wk16zj50.

[15] A. M. Agfirlana, “ANALISIS IMPLEMENTASI PERKEMBANGAN KOGNISI PIAGET DAN VYGOTSKY DALAM PENCAPAIAN TUJUAN PEMBELAJARAN PENDIDIKAN AGAMA ISLAM DI SDN MARGAASIH,” *Jurnal TAMBORA*, vol. 7, no. 1, pp. 226–234, Feb. 2023, doi: 10.36761/jt.v7i1.2178.

[16] M. Ramadanti, C. P. Sary, and S. Suarni, “PSIKOLOGI KOGNITIF (Suatu Kajian Proses Mental dan Pikiran Manusia),” *Al-Din: Jurnal Dakwah dan Sosial Keagamaan*, vol. 8, no. 1, pp. 56–69, June 2022, doi: 10.30863/ajdsk.v8i1.3205.

[17] R. Rahmawati, M. M, and M. H. Nonci, “Konstruk Teori dan Paradigma Pengetahuan,” *Socius: Jurnal Penelitian Ilmu-Ilmu Sosial*, vol. 1, no. 6, Jan. 2024, doi: 10.5281/zenodo.10514453.

[18] Y. Sudiantara, *Filsafat Ilmu Pengetahuan: Bagian pertama, Inti Filsafat Ilmu Pengetahuan*. Semarang: SCU Knowledge Media, 2020.

[19] S. Labaso, “PARADIGMA INTEGRASI-INTERKONEKSI DI TENGAH KOMPLEKSITAS PROBLEM KEMANUSIAAN,” *Al-A’raf J. Pemikir. Islam dan Filsafat*, vol. 15, no. 2, pp. 335–352, Dec. 2018, doi: 10.22515/ajpif.v15i2.1462.

[20] B. T. Pramesti and H. L. Mampouw, “Analisis Pemahaman Konsep Peluang Siswa SMP Ditinjau Dari Teori APOS,” *Cendekia*, vol. 4, no. 2, pp. 1054–1063, Nov. 2020, doi: 10.31004/cendekia.v4i2.230.

