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A Targeted Evidence Map of Learner Misconceptions and Pedagogical Interventions in Secondary Algebraic Equations and Inequalities

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Abstract. Secondary learners frequently execute algebraic procedures correctly while holding fragile conceptions of equality, variables, and order. Yet, the evidence on these misconceptions and on the interventions that address them is scattered across designs and contexts. This targeted evidence map synthesises 27 empirical reports published between 2015 and 8 September 2026 on secondary learners' reasoning about algebraic equations and inequalities, as well as related instructional interventions. Reports were located through ERIC-oriented searches, publisher platforms, DOAJ-indexed sites, and citation chaining; because platform-level search dates, query logs, export counts, and screening records were not preserved, the map does not claim exhaustive coverage or a reconstructable PRISMA flow. Study characteristics, mathematical focus, reported error patterns, intervention components, and outcome evidence were coded, and counts describe reports rather than learner prevalence. Recurrent concerns include procedural overgeneralization, equality and equivalence, signs and inverse operations, variable sense, representation coordination, prerequisite arithmetic, quadratic structure, and inequality solution sets. Multiple representations, explicit scaffolding, manipulatives, worked examples, and digital tools were reported across interventions, but designs and outcome measures are heterogeneous, and inequality-focused interventions are comparatively scarce. Technological Pedagogical Content Knowledge (TPACK) is used only as a post hoc interpretive lens because most included studies did not measure it. The findings are best read as a bounded evidence map that identifies patterns and gaps for further database-based review and primary research.

Keywords: Algebraic Equations, Algebraic Inequalities, Pedagogical Interventions, Quantitative Evidence Mapping, TPACK.

1 Introduction

Algebra is a central organising language of secondary mathematics. It enables learners to express generality, model relationships, reason about unknown quantities, connect symbolic and graphical representations, and prepare for functions, calculus, statistics, science and technology. Within this domain, equations formalise equality constraints

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while inequalities describe ordered sets of possible values. Both require learners to coordinate arithmetic knowledge with the meanings of variables, equivalence, inverse operations, symbolic structure and representation. Research has repeatedly shown, however, that successful execution of memorised procedures can coexist with fragile conceptual understanding [1]–[3].

Misconceptions in solving equations and inequalities are not merely careless slips. Some are systematic constructions that are locally sensible from a learner's previous experience. For example, an operational interpretation of the equal sign as a signal to "calculate the answer" may work for elementary arithmetic strings but fail when learners encounter identities or equations with expressions on both sides. Similarly, the classroom mnemonic "move it to the other side and change the sign" may yield correct answers for familiar linear equations while obscuring the equivalence-preserving operation that is actually performed. In inequalities, transferring equation rules without attending to order can lead to failure to reverse the inequality when multiplying or dividing by a negative quantity, or to representing a solution as a single number rather than a set [2], [4]–[6].

The contemporary intervention literature is equally heterogeneous. Explicit instruction, worked examples, balance models, algebra tiles, virtual manipulatives, GeoGebra and other digital environments, multiple representations, differentiated teaching and flipped-classroom structures have all been investigated. Reviews have reported encouraging effects for several intervention families, particularly when conceptual representations and explicit guidance are combined. Still, the strength and transferability of the evidence vary by design, population and algebraic content [7]–[9]. More recent reviews add two important points: secondary learners experiencing difficulty often have interlocking problems with equation encoding, equation solving, variables, negative numbers and prerequisite fractions, while evidence on quadratic equations remains comparatively fragmented and context-dependent [10], [11].

Technology introduces further possibilities and further demands. Dynamic graphing environments can link an equation to its graph; virtual balance models can embody equivalence; interactive worked examples can expose and repair errors; and digital resources can provide immediate variation and feedback. Yet technology does not automatically produce conceptual change. The mathematical task, teacher orchestration, learner prior knowledge, representational design and classroom infrastructure all matter. For this reason, the Technological Pedagogical Content Knowledge (TPACK) framework is useful for analysing not only whether a digital tool was present, but whether the tool was mathematically and pedagogically aligned with the misconception being targeted [12]–[14].

This review therefore treats misconceptions and interventions as two sides of the same instructional problem: what forms of reasoning break down, and what forms of teaching make the relevant mathematical structure visible, discussable and revisable? A quantitative evidence-mapping strategy is used to describe the distribution of the empirical literature without collapsing heterogeneous designs into a single summary effect. This is particularly important because the evidence includes large quasi-experiments as well as single-case, diagnostic and qualitative studies; a pooled effect size would imply commensurability that the corpus does not support.

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1.1 Purpose and research questions

The purpose of the review is to identify recurring misconceptions in secondary-school algebraic equations and inequalities, quantify how the recent empirical literature is distributed across misconception and intervention categories, synthesise the pedagogical mechanisms associated with improvement, and interpret the evidence through TPACK. The review addresses four questions:

RQ1. What misconceptions and error patterns are most frequently reported in empirical studies of secondary learners' algebraic equations and inequalities?

RQ2. What pedagogical interventions have been used to address these difficulties, and what outcome patterns are reported?

RQ3. How are content, pedagogy and technology combined across interventions, and what does TPACK reveal about the design of conceptually focused algebra instruction?

RQ4. What contextual and methodological gaps should guide future research, particularly in South African, Global South and resource-constrained settings?

1.2 Contribution of the review

The review makes four contributions. First, it integrates equations and inequalities rather than examining them as isolated topics, which makes visible how overgeneralized rules migrate from one content area to another. Second, it distinguishes study-frequency evidence from learner-prevalence evidence; a category appearing in many reports is not interpreted as a population prevalence rate. Third, it links misconception diagnosis to intervention design using TPACK, while acknowledging that TPACK is primarily a teacher-knowledge and instructional-design framework rather than a standalone theory of learner cognition. Fourth, the analysis foregrounds contextual feasibility: an intervention that requires stable broadband, one device per learner or extensive teacher preparation cannot be transferred uncritically to under-resourced classrooms.

2 Conceptual and Theoretical Framework

2.1 Misconceptions as structured but revisable reasoning

A misconception is treated here as a recurring interpretation or rule that is mathematically inappropriate beyond the conditions under which it was formed. This definition separates misconceptions from isolated slips and from errors caused only by transcription or inattention. The distinction is instructive: a slip may require checking, whereas a misconception requires reconstructing meaning. In algebra, the reconstruction often concerns relational equivalence, the status of variables, the structure of expressions, or the domain of a solution set. Skemp's distinction between instrumental and relational understanding is useful for this purpose: instrumental

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knowledge may support efficient rule execution, while relational understanding explains why a transformation preserves the solution set [3].

Representation is also central. Symbolic, graphical, tabular, verbal and number-line forms are not interchangeable displays; each foregrounds different mathematical objects and relations. Duval's account of representation emphasises that mathematical comprehension depends on coordinating transformations within and between representational registers [15]. For learning equations and inequalities, this means that a learner who can manipulate symbols but cannot connect an inequality to an interval or number line has incomplete representational control. The review therefore codes representation-coordination difficulties separately from purely procedural errors.

2.2 TPACK as the principal instructional-design lens

TPACK conceptualises effective technology integration as an interaction among content knowledge (CK), pedagogical knowledge (PK) and technological knowledge (TK), together with their pairwise intersections—pedagogical content knowledge (PCK), technological content knowledge (TCK), technological pedagogical knowledge (TPK)—and the integrated domain TPACK. In algebra, CK includes equivalence, variable meaning, inverse operations, factorisation, roots, intervals and order properties. PK includes diagnosing prior knowledge, eliciting learner reasoning, sequencing examples, scaffolding, explicit instruction, discussion, formative feedback and differentiation. TK includes graphing environments, computer algebra systems, virtual manipulatives, interactive open educational resources (OER) and learning platforms.

The value of TPACK for this review is analytical rather than celebratory. A technology-rich lesson can have weak TPACK when software merely replaces paper-based exercises. Conversely, a simple virtual balance can demonstrate strong TPACK if it is deliberately used to connect the operations performed on both sides of an equation to the invariant relation of equality. Research on algebra-focused teacher development and South African mathematics teachers also suggests that technological and technology-integrated knowledge can lag behind content and pedagogical knowledge. In contrast, time, infrastructure, professional development and beliefs constrain enactment [13], [14]. These contextual factors are shown around the core domains in Fig. 1.

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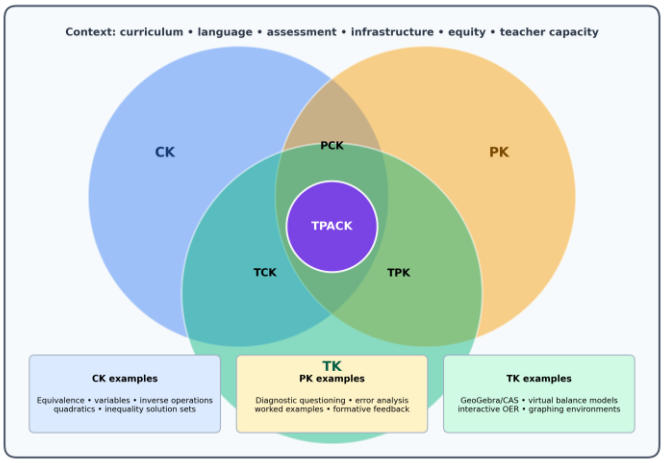


Fig. 1. Colour-coded TPACK framework adapted to the diagnosis and remediation of misconceptions in algebraic equations and inequalities.

Table 1. Operationalisation of TPACK for secondary algebraic equations and inequalities.

Domain	Meaning in this review	Algebra-specific indicators
CK	Mathematical meanings and structures	Equivalence; inverse operations; variable; factorisation; roots; zero-product property; order; intervals and solution sets.
PK	General principles of teaching and learning	Diagnostic questioning; explicit instruction; worked examples; self-explanation; formative assessment; differentiation; collaborative discourse.
TK	Capabilities and constraints of technologies	GeoGebra/graphing software; virtual balances and algebra tiles; interactive OER; CAS; graphing calculators; LMS/flipped resources.
PCK	Representing and sequencing algebra for learners	Anticipate operational equal-sign views, sign errors, illegal cancellation, factor/root confusion and inequality rule transfer; choose examples that expose contrasts.
TCK	How technology represents specific algebraic structure	Dynamically link symbolic and graphical solutions; vary parameters; show both sides of an equation; visualise intervals, intercepts and sign regions.
TPK	How technology changes pedagogy	Immediate feedback, purposeful variation, collaborative prediction, screenshot annotation, retrieval practice, differentiated task pathways.
TPACK	Integrated design for misconception repair	Select and orchestrate a tool only when its representation and feedback directly serve the targeted mathematical idea and the intended pedagogy.

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2.3 An integrated misconception-to-intervention proposition

The theoretical proposition guiding the synthesis is that durable improvement is most likely when an intervention creates a representational or explanatory contrast that makes the learner's existing rule insufficient, then supports reconstruction of the underlying relation. For instance, the instruction "subtract 5 from both sides" is conceptually stronger than "move +5 and make it −5" because it names the equivalence-preserving operation. A balance representation may then embody that relation (TCK), while teacher prompts require prediction and justification (TPK/PCK). The same principle applies to inequalities: comparing multiplication by +2 and −2 across a number line can make order reversal observable, but the teacher must explicitly connect the visual reversal to the formal property of order rather than allowing the animation to stand as unexplained evidence.

3 Method

3.1 Review design

This paper presents a targeted evidence map and narrative synthesis, not an exhaustive systematic review. The Preferred Reporting Items for Systematic Reviews and Meta-Analyses 2020 (PRISMA 2020) statement emphasised transparent eligibility and reporting; however, the source record does not contain the search exports and screening log required to report a conventional PRISMA flow. The included reports span experimental, quasi-experimental, single-case, longitudinal, diagnostic and qualitative designs. Frequencies describe the 27 reports and are not learner-prevalence estimates. Because outcomes and measures are not commensurable, no pooled effect estimate is calculated.

3.2 Search strategy, time frame and eligibility

The evidence map covers reports published from 2015 through 8 September 2026. The supplied search account refers to searches oriented toward ERIC (Education Resources Information Centre); SpringerLink, Wiley Online Library, ScienceDirect, Taylor & Francis, and SAGE publisher platforms; sites indexed in DOAJ (Directory of Open Access Journals); and backwards and forward citation tracing from recent reviews. It gives one representative Boolean structure: (secondary OR "middle school" OR "high school" OR Grade 7–12) AND ("algebraic equation*" OR "linear equation*" OR "quadratic equation*" OR inequalit* OR equivalence OR "equal sign") AND (misconception* OR error* OR difficult* OR intervention* OR instruction* OR technolog* OR manipulative* OR "worked example*"). The supplied record does not identify the exact interface or execution date for each search, retain each platform-adapted string, or preserve exported record totals. These details cannot be reconstructed from the 27-study extraction table and are not supplied here as if they had been recorded. Publisher-platform and citation searches can favour accessible outlets and

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miss studies indexed only in other databases; the English-language and access limits add further selection risk. The resulting corpus is consequently a targeted, non-exhaustive map, not a claim of global systematic coverage.

A report entered the coded corpus only when it provided empirical learner evidence from secondary school (or a separable secondary subgroup), and its data directly addressed (a) solving or interpreting equations or inequalities, (b) equivalence or algebraic structure used in such solving, or (c) a prerequisite error explicitly linked by the study to equation/inequality reasoning. A broad mathematics intervention was retained only when the relevant equation or inequality outcome was identifiable; an algebra-expression or fraction study was retained only when it documented a direct connection to the target reasoning. Eligible reports had to describe learner reasoning, errors, achievement or response to instruction, appear in English within the date window, and provide enough information for descriptive coding. Reviews and conceptual papers informed the framework but were not counted among the 27 empirical reports. Studies without a separable target outcome, postsecondary-only studies, primary arithmetic studies without a stated algebra link, and unrelated function studies were excluded.

Table 2. Eligibility criteria used for the coded evidence corpus.

Dimension	Included	Excluded
Population	Secondary/middle/high-school learners; Grades approximately 7–12	Primary-only or postsecondary-only samples without separable secondary data
Mathematical focus	Equations, inequalities, equivalence, roots/factorization, systems, or directly relevant algebraic prerequisites	Unrelated mathematics topics; functions without a separable algebra-equation component
Evidence	Empirical learner data: test, task, interview, observation, experimental outcome or single-case performance	Commentary, textbook-only analysis, conceptual papers, editorials
Publication window	2015–8 September 2026 for the coded empirical corpus	Earlier studies used only as foundational background
Language	English	Non-English reports not reliably interpretable by the review team
Outcome relevance	Misconception/error pattern or pedagogical intervention linked to learner understanding/performance	Technology description without learner evidence

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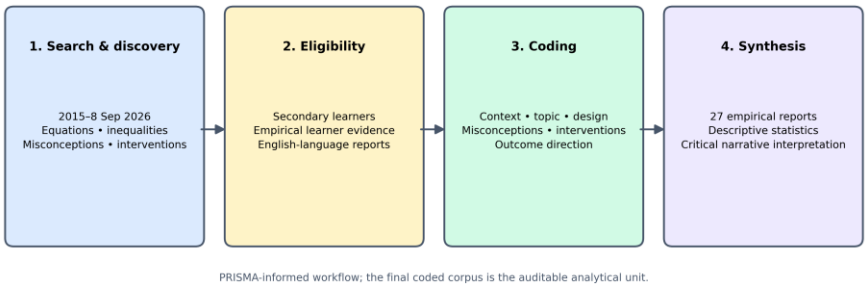


Fig. 2. Discovery and eligibility workflow for the targeted evidence map. The source record preserves the final coded corpus (n=27) but not platform-level identification totals, duplicate counts or stage-by-stage exclusion reasons; this is not a conventional PRISMA flow diagram.

3.3 Data extraction and quantitative coding

The extraction matrix recorded each report's year, country, sample/context, algebra focus, design, described intervention, reported misconception/error pattern and outcome direction. Codes were non-exclusive when a report described more than one error or intervention. Primary algebra focus was coded as the principal target to support topic summaries. For the post-hoc TPACK annotation, CK was recorded when the algebraic content target was explicit, PK when a teaching or learning strategy was described, and TK when a digital or virtual technology was named. Pairwise/integrated labels were used only where the study description explicitly linked those features; a tool's mere presence was not treated as evidence of integrated TPACK. These are analyst annotations, not primary-study TPACK measurements. The final set comprises reports from North America (11), Africa (10), and Asia (6); the absence of eligible reports from Europe, South America, and Oceania in this selected corpus does not establish that relevant research is absent from those regions. Geographic frequencies are conditional on the discovery channels, English-language restriction and access limits described above.

Intervention outcomes were separated by evidence type rather than assigned a single undifferentiated "positive" label: (1) statistically supported comparative advantage, only where the primary report identifies a supported between-group or treatment-comparison difference; (2) reported comparative direction, where an advantage is described but the available extraction does not preserve the statistic needed to verify support; (3) within-group pre/post improvement, which does not establish intervention attribution without a comparison; (4) single-case acquisition or maintenance, bounded to the participating cases; (5) mixed/null, where outcomes differ by measure or subgroup or an overall advantage is not reported; and (6) development/validation, which does not test causal effectiveness. A pre/post increase, a statistically supported between-group contrast and a single-case pattern are therefore not assigned the same evidential status. The extraction record does not preserve complete effect sizes,

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confidence intervals or p-values for all reports; where they are not reproduced in Table 3, readers should consult the primary report. No common effectiveness ranking is calculated.

3.4 Quality appraisal and interpretive weighting

The five appraisal considerations were sample/setting clarity, alignment of question and design, description of measures, transparency of analysis, and correspondence between claims and design. They were used as a narrative checklist, not as a validated scale: no numerical score, cut-off or exclusion threshold was applied, and no sensitivity analysis by score was possible. To make the limits visible, Table 3 reports the design family and a design-level evidence caution for each study. Randomised comparisons offer the strongest basis in this corpus for estimating a comparative intervention effect; quasi-experiments support comparative but potentially confounded claims; pre/post studies without a comparison support change over time but not attribution; single-case designs support case-bounded functional patterns; and diagnostic, cross-sectional or qualitative studies describe reasoning and context rather than treatment effects. Development/validation studies support feasibility or content design, not causal effectiveness. These tiers are not an overall quality ranking. As the supplied extraction record does not contain full study-level ratings or complete statistical details, conclusions are weighted by design and convergence, and the review does not claim to have completed a risk-of-bias assessment.

4 Results

4.1 Characteristics of the empirical evidence base

The final coded corpus contains 27 empirical reports. Its publication years cluster in 2018 and 2020–2025 rather than showing a steady increase. The corpus contains 11 North American reports (40.7%), 10 African reports (37.0%) and 6 Asian reports (22.2%). Seven reports are from South Africa. These proportions describe only the selected corpus; the incomplete search log and discovery-channel limitations prevent their interpretation as the global distribution of research.

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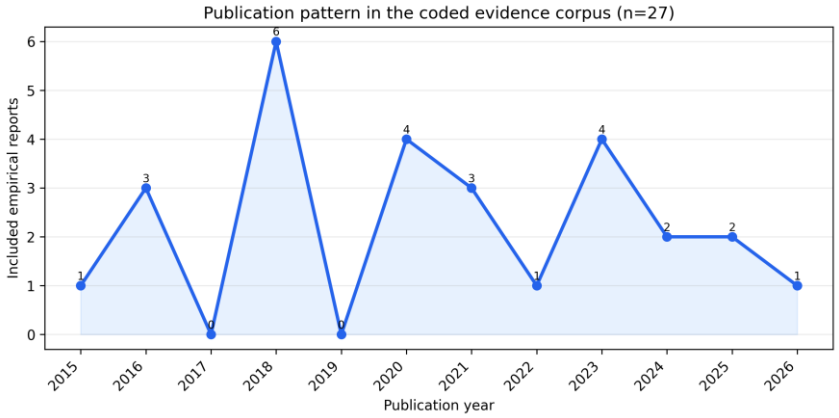


Fig. 3. Annual distribution of included empirical reports, 2015–8 September 2026 (n=27).

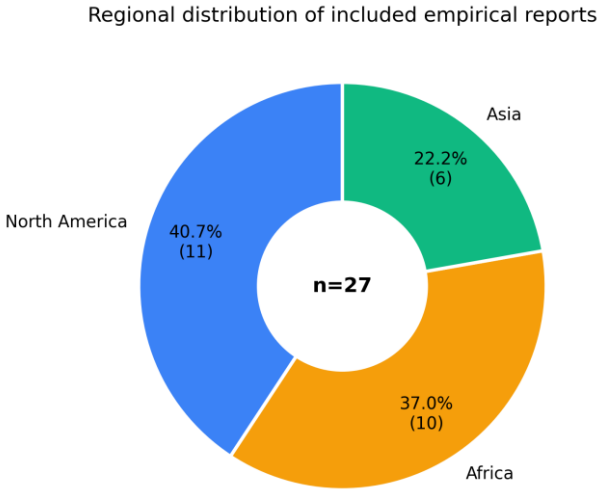


Fig. 4. Regional distribution of included empirical reports (n=27). Counts in brackets; percentages are proportions of studies, not learners.

Linear equations and equivalence dominated the corpus, but quadratic equations and inequalities formed substantial subfields. The distribution by primary topic and region (Fig. 7) shows that inequality-focused evidence was particularly sparse compared with linear-equation work. This imbalance matters because inequality reasoning introduces order properties and set-based solutions that cannot be reduced to equation-solving routines [5], [17].

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Table 3. Study-by-study synthesis matrix: sample, misconception coding, intervention, outcome evidence, design tier, post-hoc TPACK mapping and feasibility limits.

Study, setting and sample	Algebra focus, codes and intervention	Design tier and outcome evidence	TPACK mapping
Jupri et al. (2015) [18] (Indonesia); 250 Grade 7 learners (131 intervention; 119 comparison)	Linear equations and word-problem modelling. Codes: word-problem modelling; representation/translation. Design: Quasi-experimental technology-based intervention.	Tier B (comparative). Reported comparative advantage; inferential statistics not reproduced in the extraction. Technology-based initial-algebra instruction produced greater gains than comparison instruction; modelling and symbolic translation remained demanding.	CK/PK/TK analyst map; TPACK not measured.
Barbieri & Booth (2016) [19] (United States); 125 middle-school Algebra I learners	Equation solving and algebraic feature knowledge. Codes: equation solving; procedural rules/error anticipation. Design: Randomised worked-example comparison.	Tier A (randomised). Mixed evidence: subgroup benefits; full-sample effects not uniformly significant. Incorrect worked examples were especially useful for some lower-prior-knowledge learners; full-sample effects were not uniformly significant.	CK/PK analyst map; no digital tool reported.
Pournara (2016) [20] (South Africa); 250 learners tracked from Grade 9 to Grade 11	Secondary algebra errors across grades. Codes: procedural, sign and structural errors. Design: Longitudinal diagnostic study.	Tier C. Diagnostic/longitudinal evidence; no intervention outcome. Persistent errors included conjoining, negative-number/sign problems, bracket/distributive errors and weak structural understanding across grades.	Diagnostic evidence; TPACK not applicable.
Satsangi et al. (2016) [21] (United States); 3 secondary learners with learning disabilities	One-variable linear equations. Codes: linear-equation procedures/equivalence. Design: Single-case alternating-treatments study.	Tier C. Single-case accuracy improvement; evidence is limited to three learners. Both concrete and virtual manipulatives supported high accuracy in solving equations in a small single-case sample.	CK/PK + physical model; TPACK not measured.
Atteh et al. (2018) [22] (Ghana); 30	Linear equations. Codes: equivalence and equation	Tier B (pre/post). Within-group pre/post	CK/PK + balance

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Study, setting and sample	Algebra focus, codes and intervention	Design tier and outcome evidence	TPACK mapping
junior-high learners	solving. Design: Pre-test/post-test classroom intervention.	improvement; no comparison establishes attribution. Balance-model instruction was associated with pre- to post-improvement in linear-equation performance.	model; TPACK not measured.
Park & McLeod (2018) [23] (United States); 25 high-school learners with learning disabilities	Algebra concepts and procedures. Codes: broad algebra; outcome not equation-specific. Design: Comparison of multimedia OER and usual instruction.	Tier B (comparative). Mixed evidence; achievement advantage reported as small/non-significant. Multimedia OER improved some motivational/learning indicators; algebra achievement advantage was small/non-significant in the small group comparison.	Broad outcome; TPACK not measured.
Satsangi et al. (2018a) [24] (United States); 3 secondary learners with learning disabilities	Multistep linear equations. Codes: multistep equation procedures. Design: Multiple-baseline single-case intervention.	Tier C. Single-case acquisition, maintenance and generalisation. Virtual manipulatives supported the acquisition, maintenance, and generalisation of multistep equation solving among the participating learners.	CK/PK + virtual tool; TPACK not measured.
Satsangi et al. (2018b) [25] (United States); 3 secondary learners with learning disabilities	Algebraic equations. Codes: equation-solving accuracy/independence. Design: Multiple-probe single-case intervention.	Tier C. Single-case acquisition/independence improvement. Virtual manipulatives paired with explicit instruction improved accuracy and independence in equation solving.	CK/PK + virtual tool; case-bounded.
Udofia & Uko (2018) [26] (Nigeria); 500 secondary learners from six public schools	Mathematics topics including linear inequalities. Codes: linear inequalities; order/set reasoning—design: Quasi-experimental GeoGebra intervention.	Tier B (comparative). Reported comparative advantage on a broader mathematics measure; algebra-specific effect is not isolated. GeoGebra-supported instruction outperformed demonstration-oriented instruction on the mathematics achievement	CK/PK + GeoGebra; algebra effect not isolated.

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Study, setting and sample	Algebra focus, codes and intervention	Design tier and outcome evidence	TPACK mapping
		measure; linear inequality was among targeted content.	
Haskins & Hord (2018/2019) [27] (United States); 2 Grade 8 learners with learning disabilities	Individual differences in algebra tutoring. Codes: individual support needs; algebra reasoning. Design: Qualitative case study.	Tier C. Qualitative case evidence; no intervention effect estimate. Learners required different combinations of visual support, memory scaffolds and anxiety-sensitive tutoring; a uniform remediation sequence was inadequate.	Qualitative case; feasibility context-specific.
Barbieri & Booth (2020) [28] (United States); 206 middle-school learners	Equation solving and error anticipation. Codes: equation solving/error anticipation. Design: Randomised instructional experiment.	Tier A (randomised). Mixed randomised evidence; benefits vary by prior knowledge/outcome. Studying incorrect examples strengthened equation solving and error anticipation, particularly for learners with weaker prior knowledge of features.	CK/PK + error examples; no digital tool reported.
Pournara (2020) [29] (South Africa); Grade 9 diagnostic sample across school quintiles	Algebra performance and error patterns. Codes: algebra procedures, signs and structure. Design: Cross-sectional diagnostic study.	Tier C. Diagnostic evidence; no intervention outcome. Similar algebra error types appeared across school resource quintiles, highlighting both systemic and contextual dimensions of difficulty.	Diagnostic study; TPACK not applicable.
Pournara & Sanders (2020) [30] (South Africa); 106 Grade 10 learners in two township schools	Arithmetic equivalence and algebraic equations. Codes: arithmetic equivalence/equation structure. Design: Response-pattern analysis.	Tier C. Diagnostic response-pattern evidence; no intervention outcome. Success with algebraic equations did not guarantee a relational understanding of arithmetic equivalence, thereby demonstrating a dissociation between procedural and structural knowledge.	Diagnostic study; TPACK not applicable.
Zulnaidi et al. (2020) [31]	Algebraic/functional representations. Codes:	Tier B (comparative). Reported comparative	CK/PK + GeoGebra;

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Study, setting and sample	Algebra focus, codes and intervention	Design tier and outcome evidence	TPACK mapping
(Malaysia); 80 Form 2 learners (40 intervention; 40 comparison)	algebra/functional representation. Design: Quasi-experimental GeoGebra intervention.	advantage; statistics not reproduced in the extraction. GeoGebra-supported instruction yielded higher mathematics achievement than comparison instruction in the quasi-experimental sample.	broader outcome.
Birgin & Uzun Yazıcı (2021) [32] (Turkey); 52 Grade 8 learners (25 intervention; 27 comparison)	Linear equations and slope. Codes: linear equations and representation. Design: Pre/post/delayed-test quasi-experiment.	Tier B (comparative). Reported comparative advantage/retention; statistics not reproduced in the extraction. GeoGebra-supported teaching improved conceptual understanding and retention of linear equations/slope compared with comparison instruction.	CK/PK + GeoGebra; TPACK not measured.
Barbieri et al. (2021) [33] (United States); 75 Algebra I learners	Equation solving and error anticipation. Codes: equation procedures/error anticipation. Design: Worked-example/self-explanation experiment.	Tier C. Reported worked-example benefit; statistics not reproduced in the extraction. Worked examples with self-explanation improved equation learning and learners' anticipation of likely errors.	CK/PK + worked examples; no digital tool reported.
Barbieri et al. (2021) [34] (United States); 350 middle-school learners	Algebra performance profiles and fraction knowledge. Codes: fraction prerequisite/algebra profiles. Design: Predictive profile analysis.	Tier C. Predictive/profile association; not an intervention effect. Fraction knowledge predicted distinct profiles of algebra feature knowledge, encoding, equation solving and word-problem performance.	Predictive association; not causal.
Zwanch (2022) [35] (United States); 18 learners in Grades 6-9	Word problems modelled by systems of equations. Codes: systems modeling/word-problem representation. Design: Task-based qualitative interviews.	Tier C. Qualitative task evidence; no intervention effect estimate. Numerical reasoning strategies shaped how learners represented and solved word problems that could be modelled with systems of equations.	Qualitative task study; no intervention.

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Study, setting and sample	Algebra focus, codes and intervention	Design tier and outcome evidence	TPACK mapping
Egodawatte (2023) [36] (Canada); Grade 11 high-school learners	Levels of algebraic problem-solving understanding. Codes: algebraic reasoning levels. Design: Diagnostic/taxonomic analysis.	Tier C. Diagnostic taxonomy; no intervention effect estimate. Learners showed qualitatively different levels of algebraic understanding, supporting the need to diagnose structure and reasoning rather than score only final answers.	Diagnostic taxonomy; no intervention.
Mutodi et al. (2023) [17] (South Africa); 50 Grade 12 learners; 10 follow-up interviews	Linear and related algebraic inequalities. Codes: inequality sign and solution-set reasoning. Design: Diagnostic test plus interviews.	Tier C. Diagnostic evidence; no intervention outcome. Learners often treated inequalities as equations, mishandled the inequality sign and struggled to coordinate symbolic solutions with number-line/graph representations.	Diagnostic study; TPACK not applicable.
Makgakga (2023a) [37] (South Africa); Grade 11 learners	Quadratic equations by completing the square. Codes: quadratic transformations/prerequisites. Design: Newman's error-analysis study.	Tier C. Diagnostic error-analysis evidence; no intervention outcome. Completing-the-square errors occurred at multiple stages and reflected difficulties in reading, transformation, and prerequisite arithmetic.	Diagnostic error analysis; no intervention.
Makgakga (2023b) [38] (South Africa); 42 written tests; 8 learner interviews	Meaning of roots of quadratic equations. Codes: roots, factorisation and quadratic structure. Design: Diagnostic qualitative study.	Tier C. Qualitative diagnostic evidence; no intervention outcome. Learners had difficulty explaining roots, finding a missing root, solving standard-form quadratics and reconstructing an equation from given roots.	Qualitative diagnostic study; no intervention.
Baybayon & Lapinid (2024) [39] (Philippines); Grade 9 learners	Quadratic equations. Codes: quadratic equations/mastery. Design: Differentiated flipped-classroom intervention.	Tier C. Reported pre/post/mastery gains; bundled components limit attribution. Differentiated instruction within a flipped-classroom structure was associated with meaningful gains in	CK/PK + flipped format; components bundled.

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Study, setting and sample	Algebra focus, codes and intervention	Design tier and outcome evidence	TPACK mapping
		mastery of quadratic equations.	
Mirza et al. (2024) [40] (Indonesia); 6 secondary learners	Quadratic inequalities. Codes: quadratic inequality/solution set. Design: Diagnostic case study.	Tier C. Diagnostic case evidence; no intervention outcome. Learners displayed misconceptions in quadratic inequalities regarding algebraic manipulation, critical values, and the interpretation of the solution set.	Diagnostic case; n=6; not prevalence evidence.
Stemele & Asvat (2025) [41] (South Africa); 22 Grade 9 learners	Algebraic expressions as prerequisites to equation solving. Codes: expression structure prerequisite. Design: 10-day algebra-tiles pre/post intervention.	Tier B (pre/post). Pre/post improvement; no comparison establishes attribution. Algebra-tile instruction reduced several expression-related errors and improved overall performance, illustrating the value of structural representations before working with equations.	CK/PK + physical tiles; TPACK not measured.
Abdussalam et al. (2025) [42] (Nigeria); 361 senior-secondary learners (182 intervention; 179 comparison)	Quadratic equations. Codes: quadratic structure/performance. Design: Quasi-experimental algebra-tiles intervention.	Tier B (comparative). Reported comparative advantage and retention; statistics not reproduced in the extraction. Algebra-tile instruction was associated with higher performance and retention on quadratic equations than comparison instruction in a large quasi-experimental study.	CK/PK + physical tiles; TPACK not measured.
Salazar & Uy (2026) [43] (Philippines); 60 Grade 9 learners plus 3 evaluators	Quadratic equations and inequalities. Codes: quadratic equations and inequalities. Design: Development/validation of strategic intervention package.	Tier D. Development/validation; no causal effectiveness test. A strategic intervention package was developed around least-mastered quadratic equation/inequality competencies; the study primarily supports	Development/validation; not effectiveness evidence.

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Study, setting and sample	Algebra focus, codes and intervention	Design tier and outcome evidence	TPACK mapping
		design/validation rather than causal effectiveness.	

Matrix note. Design tiers are descriptive limits on inference, not a validated quality score. TPACK labels are post-hoc mappings of features described in this manuscript; no primary study is represented as having measured TPACK unless its source did so. Numerical effect estimates and p-values are not reconstructed where the extraction record does not preserve them. Contextual feasibility is marked only where it is reported; missing context is not assumed. Tier A = randomised comparison; Tier B = comparative/quasi-experimental or pre/post without comparator; Tier C = diagnostic, qualitative, or single-case; Tier D = development/validation. Contextual feasibility was not reported in the extraction for any of the 27 studies.

4.2 Recurring misconceptions and error patterns

Figure 5 counts reports in which each non-exclusive misconception/error category was represented. The ranking is descriptive of coding frequency, not of learner prevalence, severity or instructional importance. A category that recurs across small case studies and one that appears in a large diagnostic sample contribute equally to the report count. Table 3 retains the sample information study by study so readers can see this variation; no category-specific learner total is calculated because the source extraction does not preserve a verified study-by-category cross-tabulation and the diagnostic instruments are not equivalent. A learner-weighted total would therefore imply comparability that this corpus cannot support.

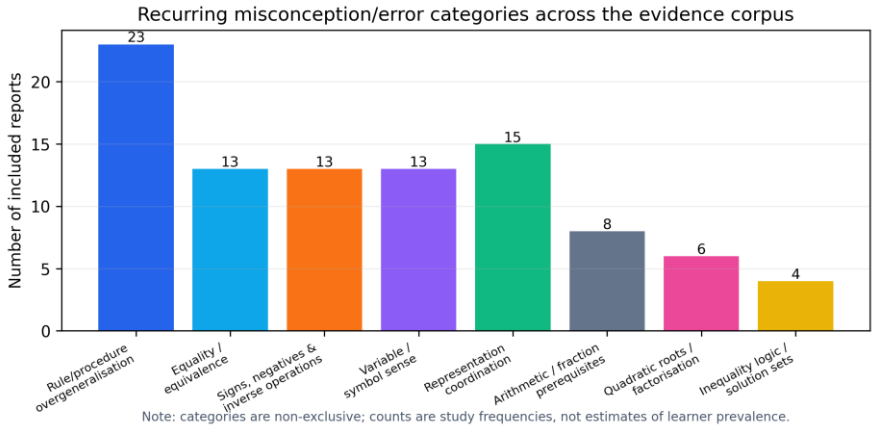


Fig. 5. Number of reports coded for each non-exclusive misconception/error category. Counts are study frequencies; they are not learner-prevalence estimates, severity rankings or sample-size-weighted totals.

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Table 4. Misconception typology, illustrative errors and pedagogical responses.

Category	Mathematical interpretation	Illustrative learner response	Pedagogical response
Operational equality/equivalence	Interpreting "=" as "calculate/write the answer"; treating two sides as sequential rather than equivalent expressions.	In $3x + 5 = 20$, learner says "move 5" without recognising that subtracting 5 from both sides preserves equality.	Use balance/relational language; compare true/false open sentences; require operations to be stated on both sides.
Illegal conjoining / structural collapse	Combining unlike terms or collapsing an expression because arithmetic experience suggests every expression should become one result.	$2x + 3 \rightarrow 5x$; or $x + x^2 \rightarrow 2x^3$.	Use substitution checks, algebra tiles and counterexamples; explicitly distinguish terms, coefficients and powers.
Sign and inverse-operation errors	Losing negative signs, changing a sign because a term "crosses" the equal sign, or misusing subtraction of a negative.	$x - (-3) = 7$ treated as $x - 3 = 7$.	Represent the actual inverse operation; use number-line/zero-pair reasoning and deliberate contrast cases.
Variable and symbol-sense misconceptions	Treating a letter as a label/object, a fixed hidden number in every context, or ignoring its structural role.	$3a + 2b$ interpreted as $5ab$; x in two equations assumed to have the same value regardless of context.	Vary meanings of variables; connect verbal, tabular and symbolic representations; discuss parameter versus unknown roles.
Distribution and brackets	Applying multiplication to only the first term or inventing distribution across non-distributive operations.	$2(x + 3) = 2x + 3$.	Use area/algebra-tile models and substitution checks; contrast $2(x+3)$ with $2x+3$.

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Category	Mathematical interpretation	Illustrative learner response	Pedagogical response
Quadratic roots and factorisation	Confusing factors with roots; not using the zero-product property; incomplete square or factorisation procedures.	$(x-2)(x+5)=0$ gives roots 2 and 5 rather than 2 and −5.	Connect factors, intercepts and roots graphically; verify by substitution; compare factoring, completing the square and formula methods.
Inequality sign reversal	Applying equation rules unchanged when multiplying/dividing by a negative quantity.	$-2x > 6$ gives $x > -3$ instead of $x < -3$.	Use ordered-number examples and number-line transformations to show reversal; explicitly state the order property.
Inequality solution set	Reporting one value rather than an interval/set or misreading open/closed endpoints.	$x \geq 4$ reported as $x=4$ only.	Coordinate symbolic, interval and number-line forms; test boundary and interior/exterior values.
Prerequisite arithmetic/fraction gaps	Weak knowledge of integers, fractions, ratios, or distribution overloads algebraic reasoning.	$x/3 + 1/2 = 4$ solved by manipulating denominators inconsistently.	Diagnose prerequisites; interleave fraction/negative-number repair with algebra rather than treating it as unrelated remediation.

Equality and equivalence as a foundational bottleneck. The coded evidence and foundational research converge on equality as a major organising concept. Learners who interpret "=" operationally can perform one-step routines yet struggle when expressions appear on both sides, when an equation is presented in non-canonical form, or when they are asked to judge equivalence without calculating. South African evidence is especially informative because response-pattern studies show that equation success and arithmetic-equivalence success do not perfectly coincide [6], [30]. This means teachers should not infer relational understanding from correct answers alone. Diagnostic tasks such as $8 + 4 = \square + 5$, or asking whether $3(x+2)=3x+6$ is "the same value for every x", can reveal the underlying concept more directly.

Sign, negative-number and inverse-operation reasoning. Sign-related errors appeared across linear, quadratic and inequality contexts. The same superficial rule can manifest differently: changing a term's sign when "moving" it, losing a negative sign during factorisation, or failing to reverse an inequality after dividing by a negative number. These are related because each depends on distinguishing a written sign from the operation or order relation it represents. The instructional implication is that sign

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rules should be justified through operations and structure rather than taught as isolated mnemonics. Where manipulatives are used, they should be connected explicitly to symbolic notation; where graphs or number lines are used, they should be connected back to formal algebraic statements.

Quadratic equations and inequalities. Recent evidence on quadratic equations identifies difficulties with factoring, completing the square, understanding roots and coordinating multiple solution methods [11], [37], [38], and the work of Star and Rittle-Johnson [44] on flexibility in equation solving indicates that comparing solution methods helps learners connect them. A major conceptual issue is that methods are often learned as disconnected recipes. Factoring, graphical roots and the zero-product property should instead be connected: if $(x-2)(x+5)=0$, the roots $x=2$ and $x=-5$ are precisely the x-intercepts of $y=(x-2)(x+5)$. Quadratic inequalities add another layer because the solution is usually an interval or union of intervals determined by the sign of the quadratic expression, not simply its roots [40].

Inequality-specific conceptual demands. Inequality research remains smaller than equation research in the corpus, but it shows distinct conceptual demands. Learners may treat an inequality as an equation with a different symbol, operate on both sides without considering the order of operations, or stop once boundary values are found. Studies of inequalities have long documented inappropriate analogies with equation solving [5], and recent South African evidence confirms difficulties with inequality signs, multiplication by variables and graphical/number-line representation [17]. Effective instruction therefore needs to foreground three ideas: order is preserved under addition/subtraction of the same quantity; order reverses under multiplication/division by a negative quantity; and the solution is a set that must be represented and tested.

4.3 Pedagogical interventions and reported outcome patterns

The intervention components in the coded reports were frequently combined. Multiple representations, digital/dynamic tools, explicit or scaffolded instruction, manipulatives, and worked-example/error-analysis routines appeared in different combinations. This map records reported presence, not isolated component effects. A study combining several elements cannot establish which component caused an outcome unless its design separately tests those components. Reported outcome direction is therefore interpreted alongside study design and sample, rather than as a common effectiveness ranking.

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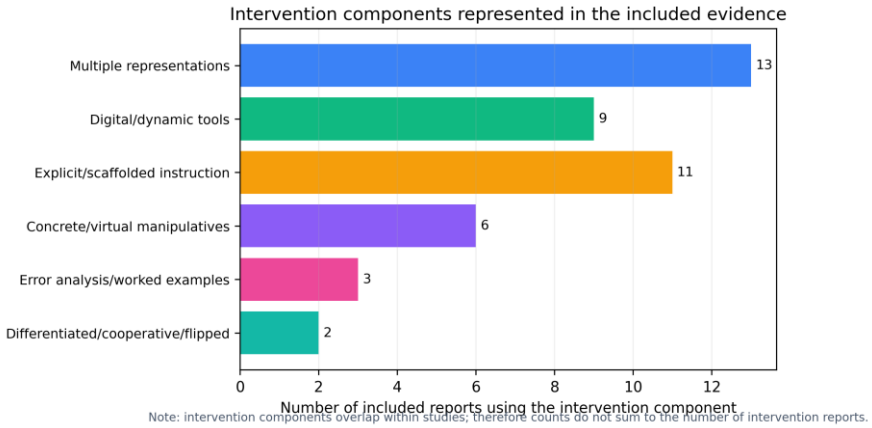


Fig. 6. Frequency of pedagogical intervention components in the coded corpus. Components overlap within studies.

Table 5. Intervention families, mechanisms, evidence pattern and cautions.

Intervention family	Proposed mechanism	Evidence pattern in corpus	Caution for interpretation
Balance models and algebra tiles	Makes equivalence, zero pairs, distribution and factor structure visible.	Pre/post change or case-level improvement; evidence types remain distinct in linear and quadratic contexts; transfer depends on explicit connection to symbols.	Avoid leaving learners at a concrete picture level; require symbolic justification and gradual fading of the model.
Virtual manipulatives	Combines visual structure with repeatable interaction and immediate change.	Small single-case studies with learners with learning disabilities reported strong acquisition/maintenance patterns.	Evidence is narrow in sample size; usability and teacher prompting matter.
GeoGebra / dynamic graphing	Coordinates, equations, graphs, parameter variation, intercepts and intervals.	Quasi-experimental studies frequently report achievement/conceptual gains; evidence of retention is promising in some settings.	Technology is not a substitute for explanation; ensure tasks require prediction, comparison and justification.

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Intervention family	Proposed mechanism	Evidence pattern in corpus	Caution for interpretation
Worked examples and error analysis	Externalises solution steps and makes common wrong rules objects of discussion.	Randomised studies show benefits for equation solving/error anticipation, sometimes stronger for learners with lower prior knowledge.	Incorrect examples must be clearly resolved; avoid increasing confusion through poorly scaffolded error exposure.
Explicit/scaffolded instruction	Reduces working-memory load and makes reasoning steps, vocabulary and checks overt.	Common in successful interventions and systematic reviews, especially for learners experiencing difficulty.	Explicit instruction should teach meaning and decision criteria, not only procedural scripts.
Differentiated/flipped/cooperative structures	Uses class time for guided problem solving while adapting pathways to readiness.	Recent studies on quadratic equations report gains, but intervention packages combine many components.	Component-specific causal attribution is difficult; access to devices/data may affect equity.
Prerequisite repair and interleaving	Targets integer, fraction and arithmetic equivalence knowledge that predicts algebra success.	Profile and diagnostic studies show strong links between prerequisite knowledge and later algebra performance.	Remediation should remain connected to algebraic purpose rather than delaying algebra indefinitely.

Multiple representations are beneficial when connections are made explicit. The repeated presence of multiple representations should not be interpreted as evidence that adding pictures automatically improves learning. Diagrams and digital displays can impose additional representational demands if learners do not understand what each element means [15], [45]. The stronger interventions use representation as a bridge: a balance model is linked to identical operations on both sides; algebra tiles are linked to coefficients and zero pairs; a graph is linked to roots or boundary values; a number line is linked to an interval statement. The pedagogical work lies in the translation among representations.

Worked examples and productive use of errors. Worked-example research in the corpus suggests that incorrect solutions can be instructionally valuable when learners are required to locate, explain and correct the error [19], [28], [33]. This approach directly targets misconceptions by making the wrong rule visible. It also supports metacognitive monitoring: learners learn not only a correct method but what a plausible

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wrong method looks like. The evidence is nuanced, however; benefits can depend on prior knowledge and task design. Teachers should therefore select high-frequency, conceptually diagnostic errors rather than presenting a catalogue of arbitrary mistakes.

Manipulatives and balance representations. The balance model has an intuitive fit with equation equivalence, and the systematic review by Otten et al. [8] shows that it is widely used. However, the empirical base varies in quality. In the present corpus, concrete/virtual balance approaches and algebra tiles were associated with improved performance in several studies [21], [22], [24], [25], [41], [42]. Their strongest conceptual role is not "making algebra easier" in a general sense, but embodying invariants: equal quantities remain equal under the same operation; zero pairs represent additive inverses; rectangular area can represent product structure. The representational model must eventually be faded so that learners can reason flexibly with symbols.

Digital and dynamic technologies. The digital interventions described in the corpus include technology-based initial algebra environments, GeoGebra, multimedia OER, and virtual manipulatives. Comparative, pre/post and single-case findings differ in design and strength; their directions are summarised in Table 3 without treating them as equivalent. TPACK helps interpret how a reported tool may relate to mathematical content and teaching. Still, the present mapping is post hoc: tool use and instructional features were coded from study descriptions, whereas CK, PK, TK, and their intersections were not measured consistently across the primary studies. The map therefore reports evidence about the intervention as described, followed by an analyst's TPACK interpretation; it does not claim that the included studies demonstrated or tested TPACK.

4.4 Evidence by algebra topic and region

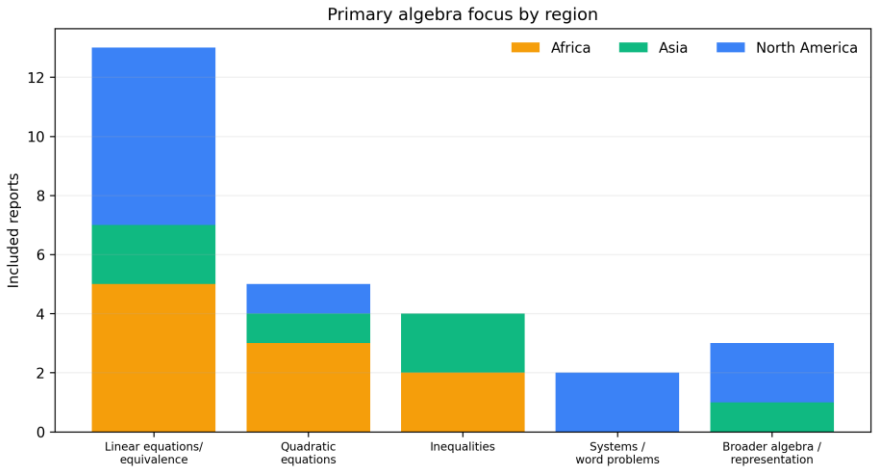


Fig. 7. Primary algebra focus of the included reports by region (exclusive topic coding; n=27).

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Figure 7 shows that linear equations/equivalence are more frequently represented than inequalities in this corpus. The selected African reports are concentrated in South Africa and Nigeria. Because this distribution is conditioned by incompletely logged search channels and language and access restrictions, it should be read as a gap in this map rather than as proof of a global research gap. Curriculum sequencing, language, class size, infrastructure, and assessment may shape both learners' responses and the feasibility of intervention; cross-context transfer requires direct study rather than assumptions.

5 Discussion

5.1 Misconceptions are relational failures, not simply missing rules

Across the evidence, the most important pattern is that many errors are rational from within a restricted rule system. "Change the sign when it crosses the equals sign" is a compressed record of an inverse operation; "the answer comes after the equals sign" is a schema built from years of arithmetic tasks; "solve an inequality like an equation" transfers a recently learned algorithm. These rules fail because their domain of validity is not understood. Teaching additional rules can therefore compound the problem. The more durable response is to identify the invariant relation—equivalence, product equals zero, or order—and show how each procedure follows from it.

This interpretation also explains why prerequisite knowledge matters. Fractions, integers and distributive structure are not separate background topics; they are the material through which algebraic transformations operate. A learner solving $x/3 + 1/2 = 4$ must coordinate equivalence, fractional arithmetic and inverse operations simultaneously. Profile studies and recent reviews show that weak fraction and arithmetic knowledge can predict or accompany algebra difficulty [10], [34]. Diagnostic teaching should therefore identify which layer is failing rather than labelling the entire response as an "algebra problem".

5.2 Pedagogical interventions work through mechanisms, not brands

The intervention synthesis suggests that no single resource—GeoGebra, algebra tiles, a balance scale, a flipped lesson or a worked-example worksheet—should be treated as an intervention mechanism by itself. The mechanism is the change in learner activity and representation that the resource enables. Algebra tiles can make distribution visible, but only if learners map tile area/length conventions to symbolic coefficients. GeoGebra can show that roots are x-intercepts, but only if learners interpret the graph and connect it to factorisation. Incorrect worked examples can strengthen monitoring, but only if learners explain why a step is invalid and what mathematical property has been violated.

This finding is consistent with the earlier algebra meta-analysis, which found positive effects across several broad instructional categories and larger average effects for conceptually focused than exclusively procedural approaches [9]. It is also

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consistent with the recent review of quadratic equations, which found promise in geometry-aided and technology-based teaching while warning that the evidence remains limited and heterogeneous [11]. The appropriate conclusion is therefore not that technology "works", but that interventions are strongest when they coordinate conceptual structure, carefully sequenced examples, explicit explanation, learner reasoning and representational support.

5.3 A TPACK interpretation of the intervention evidence

TPACK is used here as an interpretive organising framework, not as the principal measured construct of the included evidence. The study-by-study table identifies the reported content focus, pedagogical approach, and technology as described in the manuscript. The CK/PK/TK tags are analyst-applied descriptions of those reported features; intersections such as TCK, TPK and integrated TPACK are not inferred unless the study description explicitly supports the relationship. Even then, the tags indicate a plausible instructional alignment, not a validated measure of teacher knowledge or an outcome tested by the primary study. This distinction prevents a post-hoc theoretical reading from being mistaken for evidence generated by the original research design.

For inequalities, TPACK can support a particularly important representational transition. CK identifies the order principle; TCK identifies a dynamic number line or graph that can show how multiplying coordinates by −1 reverses order; PCK anticipates that learners will transfer equation rules; and TPK structures a task in which learners predict before manipulating the display, then explain why the direction changes. The technology becomes part of a reasoning sequence rather than an animated answer key.

5.4 Context, equity and feasibility

The regional distribution in this targeted corpus cannot support universal claims about intervention transfer. Device access, data costs, electricity reliability, class size, language and teacher learning affect implementation. The South African studies included here raise context-specific concerns, but the limited search record does not establish their representativeness of the country or region. Technology feasibility statements in Table 3 are drawn from the contextual information recorded in the manuscript; where infrastructure was not reported, feasibility is marked as not documented rather than assumed.

Equity-oriented implementation does not imply abandoning technology. It implies selecting technologies according to instructional value and feasibility. One projected GeoGebra display, a teacher-operated virtual balance, reusable algebra tiles, printed worked-example cards, or offline interactive activities can preserve the conceptual mechanism without requiring one-device-per-learner connectivity. TPACK should therefore be understood as context-sensitive design knowledge, not as a hierarchy in which more technology is necessarily better.

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6 Implications for Practice, Teacher Education and Research

6.1 Classroom implications

a. Diagnose before reteaching. Use short tasks that reveal the meanings of equality, sign, variable, and solution set, not only final-answer accuracy.
b. Replace transposition mnemonics with equivalence-preserving language. Learners should name the operation applied to both sides and justify why the solution set is unchanged.
c. Use deliberate contrast. Pair examples that look similar but require different reasoning, such as $2x>6$ and $-2x>6$, or $(x-2)(x+5)=0$ and $(x-2)(x+5)=7$.
d. Coordinate at least two representations when the second representation exposes structure: equation–balance, factorisation–graph, inequality–number line, symbolic solution–interval notation.
e. Require verification. Substitution, boundary testing and graphical checks turn answers into claims that can be evaluated.
f. Fade supports. Move from concrete/visual representations to diagrams, and then to flexible symbolic reasoning, so learners do not become dependent on a model.
g. Treat errors as data. Select common wrong solutions for explanation and correction, while maintaining a classroom norm that mathematical errors are objects for analysis rather than evidence of low ability.

6.2 Teacher education and professional development

Teacher education should connect algebra-specific PCK with technology-specific representations. Generic software training is insufficient. A TPACK-oriented workshop can begin with a misconception—for example, failure to reverse an inequality after division by a negative—and ask teachers to (1) articulate the mathematical principle, (2) anticipate learner reasoning, (3) select a representation or digital tool, (4) design questions that make the principle visible, and (5) plan a low-technology alternative for constrained settings. This structure directly integrates CK, PCK, TCK and TPK rather than teaching them as isolated competencies.

Professional development should also build teachers' diagnostic repertoire. Teachers need examples of wrong but plausible solutions, knowledge of likely prerequisite sources, and strategies for distinguishing a slip from a stable misconception. Where digital tools are introduced, teacher learning should include task design, classroom orchestration and interpretation of learner responses, not only technical operation [13], [14].

6.3 Curriculum and assessment implications

Curricula should sequence algebra so that relational equivalence, negative-number reasoning, distributive structure and fraction operations are revisited within equation contexts rather than assumed to be secure. Assessment should include explanation,

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comparison and representation tasks, not only routine solving. For inequalities, assessment should explicitly test boundary values, interval/number-line representation and reasoning about negative multiplication. For quadratic equations, tasks should connect factorisation, roots, graphs and alternative solution methods. Such assessment creates incentives for relational understanding and exposes misconceptions that procedural items can mask, consistent with the emphasis on reasoning and sense-making in the Principles to Actions of the National Council of Teachers of Mathematics [46].

6.4 Research agenda

Table 6. Priority research gaps derived from the synthesis.

Gap	Why it matters	Recommended direction
Inequality-specific interventions	Far fewer intervention studies target inequalities than linear equations.	Conduct controlled and design-based studies focused on sign reversal, interval reasoning and symbolic–number-line coordination.
Longitudinal retention and transfer	Many interventions use immediate post-tests.	Include delayed post-tests and transfer tasks with unfamiliar equation/inequality forms.
Component analysis	Interventions often bundle technology, explicit teaching and differentiation.	Use factorial or dismantling designs where feasible to identify active components.
Teacher TPACK linked to learner outcomes.	TPACK is often measured through teacher self-report rather than linked to changes in learner misconceptions.	Use observation/task analysis, along with learner diagnostics, to test whether algebra-specific TPACK predicts instructional quality and conceptual gain.
Multilingual and resource-constrained classrooms	Evidence is concentrated in a limited set of countries and well-defined research settings.	Study low-bandwidth/offline, shared-device and multilingual designs in rural and high-enrollment schools.
Measurement of misconceptions	Studies use non-equivalent tests and inconsistent terminology.	Develop validated diagnostic item banks that distinguish slips, procedural errors, and conceptual misconceptions.
Open/reproducible synthesis	Many primary studies do not share instruments or analyzable data.	Publish item sets, coding rubrics and de-identified outcome data where ethics and policy permit.

7 Limitations

This evidence map has limitations that constrain its claims. First, the search account identifies ERIC-oriented searches, named publisher platforms, DOAJ-indexed sites and

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citation chaining. Still, the supplied record does not preserve exact database/interface execution dates, complete platform-specific Boolean strings, export files, duplicate counts or stage-by-stage screening decisions. A conventional PRISMA flow and a reproducible completeness claim cannot be reconstructed; Figure 2 is therefore labelled as a discovery workflow, and the corpus is described as targeted rather than exhaustive. Second, English-language eligibility, access constraints and publisher-oriented discovery may have omitted relevant studies and may help explain the observed geographic distribution. Third, the non-exclusive error categories depend on judgment because primary studies use overlapping labels. The supplied manuscript does not report independent duplicate coding, an inter-rater statistic or a formal adjudication log; category counts are consequently exploratory and should not be interpreted as reliability-tested ranks. Fourth, the study designs, instruments and sample sizes vary widely. Report counts give each study equal weight, and no category-specific learner coverage is reported because a verified study-by-category crosswalk and comparable measures are unavailable in the extraction record. Fifth, the five quality considerations were applied narratively, without a validated scale, scores, threshold or sensitivity analysis. The evidence tiers in Table 3 communicate design limits; they are not risk-of-bias scores. Sixth, intervention outcome directions are separated by comparison, pre/post, single-case, mixed/null and development evidence; effect estimates are not pooled. Finally, most primary reports did not measure TPACK. Its codes on this map are post hoc interpretive annotations, not evidence-derived measures of teacher knowledge. These constraints mean the synthesis can identify patterns in the reports located but cannot establish global coverage, learner prevalence, uniform intervention effectiveness, or measured TPACK relationships.

The next review should search a pre-specified set of bibliographic databases (including ERIC and suitable multidisciplinary education indexes), save database-specific dates and complete search strings, export and deduplicate records, and report each screening stage with exclusion reasons. At least two reviewers should independently screen and code a sample, resolve disagreements using a documented procedure and report agreement with an appropriate statistic. A full-text study matrix should preserve learner counts, instruments, statistical results, design-based appraisal and contextual feasibility. Until those records are available, the findings in this paper should be read as a targeted evidence map and critical synthesis of 27 located reports, not as exhaustive global coverage.

8 Conclusion

This targeted evidence map of 27 empirical reports indicates that secondary learners' difficulties with equations and inequalities often involve meaning, structure, and representation, as well as procedural execution. The most recurrent coded categories concern procedural overgeneralization, equality/equivalence, signs and inverse operations, variable/symbol sense, representation coordination, prerequisite arithmetic/fractions, quadratic structure and inequality solution sets. These are report

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frequencies in a bounded corpus, not estimates of how common the misconceptions are among learners.

Intervention reports describe multiple representations, explicit scaffolding, concrete or virtual manipulatives, error analysis, worked examples and digital environments. Their outcomes differ by design, sample, measure, and duration; comparative advantage, pre/post improvement, case-level change, and resource development are not equivalent forms of evidence. The synthesis therefore supports a cautious instructional implication: representations and tools are most defensible when learners are asked to explain the mathematical relation they expose and when the teacher checks transfer to symbolic reasoning. TPACK offers a useful post hoc vocabulary for describing that alignment, but it was not measured in most studies. A database-based, fully logged review with independent screening/coding and complete study-level evidence remains necessary before making stronger claims about coverage, prevalence or relative effectiveness.

DECLARATIONS

Author Contributions

The sole author was responsible for conceptualisation, methodology, investigation, analysis, writing, review, and approval of the manuscript.

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Conflict of Interest

The author declares that no competing financial, professional or personal interests exist that could have influenced the work reported in this manuscript.

Data Availability Statement

The coded study characteristics and descriptive counts are reported in Table 3 and the figures. No individual-participant data were collected. The underlying search exports and stage-by-stage screening log are not available in the record used for this revision.

Artificial Intelligence (AI) Disclosure

ChatGPT (OpenAI) and Grammarly were used to support editorial restructuring, language revision and citation formatting. The author reviewed the AI-assisted changes and remains responsible for the accuracy and final content of the manuscript.

Research Ethics: Ethical review and approval were not required because this evidence map analyses published studies and did not involve direct participation by human subjects.

Informed Consent Statement

Not applicable; no human participants were recruited for this evidence map.

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